Inefficient frequency was first noticed in [1]. Here we have a more general model in which different frequency can have different idle power.

1 What is inefficient frequency?

A frequency $f_1$ is inefficient with respect to a higher frequency $f_2$ if for any task, running at frequency $f_1$ consumes more energy than running at frequency $f_2$. This means that running at lower frequency results in longer execution time and more energy consumption. Therefore, we should never use inefficient frequency in the system.

2 What is the sufficient and necessary condition for inefficient frequency?

**Theorem 1** Frequency $f_1$ is inefficient with respect to frequency $f_2$ ($f_1 < f_2$) iff

$$\frac{P(f_1) - I(f_2)}{f_1} \geq \frac{P(f_2) - I(f_2)}{f_2}.$$ 

**Proof:** (simplified version) Let the computational requirement of the task be $C$ cycles. If frequency $f_1$ is used to execute the task, then the execution time is $\frac{C}{f_1}$, and the energy consumption is $\frac{C \cdot P(f_1)}{f_1}$. If frequency $f_2$ is used, the execution time is $\frac{C}{f_2}$ which is less than $\frac{C}{f_1}$. Thus, the CPU has to idle for $\frac{C}{f_1} - \frac{C}{f_2}$ time during which it consumes idle power $I(f_2)$. Therefore the energy consumption when using frequency $f_2$ is $\frac{C \cdot P(f_1)}{f_1} + I(f_2)\left(\frac{C}{f_1} - \frac{C}{f_2}\right)$.

If $f_1$ is inefficient with respect to $f_2$, then we have

$$\frac{C \cdot P(f_1)}{f_1} \geq \frac{C \cdot P(f_2)}{f_2} + I(f_2)\left(\frac{C}{f_1} - \frac{C}{f_2}\right),$$

which can be arranged as

$$\frac{P(f_1) - I(f_2)}{f_1} \geq \frac{P(f_2) - I(f_2)}{f_2}.$$ 

3 The transitivity of efficient frequencies

**Theorem 2** If frequency $f_1$ is efficient with respect to $f_2$ and $f_2$ is efficient with respect to $f_3$ ($f_1 < f_2 < f_3$), then $f_1$ is efficient with respect to $f_3$.

**Proof:** Since $f_1$ is efficient with respect to $f_2$, we have

$$\frac{P(f_2) - I(f_2)}{f_2} > \frac{P(f_1) - I(f_2)}{f_1} \quad \left( \Rightarrow \frac{f_2}{P(f_1) - I(f_2)} > \frac{f_1}{f_2} \right).$$
and because \( f_2 \) is efficient with respect to \( f_3 \), we have

\[
\frac{P(f_3) - I(f_3)}{f_3} > \frac{P(f_2) - I(f_2)}{f_2}
\]

and continue from the above

\[
> \frac{P(f_1) - I(f_2) + \frac{f_1}{f_3}(I(f_2) - I(f_3))}{f_1} > \frac{P(f_1) - I(f_3)}{f_1}
\]

which proves the claim. □

References